

# Fast control of the reflection of a ferroelectric by an extremely short pulse

J.-G. Caputo<sup>1</sup>, A.I. Maimistov<sup>2,3</sup> and E.V. Kazantseva<sup>2,4</sup>

<sup>1</sup>: *Laboratoire de Mathématiques, INSA de Rouen,  
Avenue de l'Université,*

*Saint-Etienne du Rouvray, 76801 France*

<sup>2</sup>: *Department of Solid State Physics and Nanostructures,  
Moscow Engineering Physics Institute,  
Kashirskoe sh. 31, Moscow, 115409 Russia*

<sup>3</sup>: *Department of General Physics,  
Moscow Institute for Physics and Technology,  
Institutskii lane 9, Dolgoprudny,  
Moscow region, 141700 Russia*

<sup>4</sup>: *Department of Condensed Matter Physics,  
Moscow Institute of Radiotechnics,  
Electronics and Automation,  
Vernadskogo pr. 78, Moscow, 119454 Russia  
E-mails: elena.kazantseva@gmail.com,  
caputo@insa-rouen.fr, aimaimistov@gmail.com*

(Dated: October 25, 2012)

We propose a new type of optical switch based on a ferroelectric. It is based on the gap which exists for waves propagating from a dielectric to a ferroelectric material. This gap depends on the polarization of the ferroelectric. We show that it can be shifted by a control electromagnetic pulse so that the material becomes transparent. This device would shift much faster than the relaxation time of the ferroelectric (1 nano s). Estimates are given for a real material.

PACS numbers: Ferroelectric materials, 77.84.-s ,Switching in ferroelectrics, 77.80.Fm Optical switches, 42.79.Ta

## I. INTRODUCTION

The rapid control of light is a problem that has been studied for a long time, in particular because of the applications. Ferroelectric materials are good candidates for this control because they can be activated using an electric field. The principle is the following. The reflection and transmission properties of a ferroelectric material depend on its state of polarization, in particular it's spontaneous polarization which exists in the absence of electric field. The controlling electric field will shift this polarization so that then we can control the reflection and transmission of any electromagnetic wave.

A first idea is to use a constant electric field such as in [1]. However then we need to take into account the relaxation of the ferroelectric which is about one nanosecond and this limits considerably the possibilities for applications. Another idea is to use a fast electromagnetic pulse so that the relaxation of the ferroelectric can be neglected. Here there are two frequency regions, one is the high frequency signal we want to control, the other, of lower frequency is the control signal. To fix ideas we choose the high frequency signal to be about a femtosecond in period and the control signal ten times slower. The time scales are such that we can neglect the relaxation of the polarization. This way we achieve light control with light, using an ultra-short pulse. In this article, we show that the reflection coefficient is equal to one in

a frequency region, a "gap" whose position depends on the state of the polarization and the control. Shifting the control, we move the gap and render the ferroelectric opaque or transparent to femtosecond waves. After deriving the model equations for the field and polarization, we linearize them around a functioning point and derive the reflection and transmission coefficients using a scattering formalism. These are then discussed as a function of the control to show how light can be driven by the femtosecond pulse.

## II. BASIC EQUATIONS

The Maxwell equations in a dielectric medium are taken in the following form

$$\begin{aligned}\nabla \times \mathbf{E} &= -\mathbf{B}_{,t}, & \nabla \cdot \mathbf{E} &= -\epsilon_0^{-1} \nabla \cdot \mathbf{P}, \\ c^2 \nabla \times \mathbf{B} &= \mathbf{E}_{,t} + \epsilon_0^{-1} \mathbf{P}_{,t}, & \nabla \cdot \mathbf{B} &= 0,\end{aligned}$$

where the subscripts indicate the time derivative. From these equation the wave equation results in

$$c^2 \nabla^2 \mathbf{E} - \mathbf{E}_{,tt} = \epsilon_0^{-1} [\mathbf{P}_{,tt} - \nabla(\nabla \cdot \mathbf{P})].$$

The geometry is shown in Fig. 1, it is a dielectric layer for  $z < 0$  and a ferroelectric layer for  $z \geq 0$ . We assume the electric field to be polarized along  $x$ . The spontaneous polarization of the ferroelectric is supposed

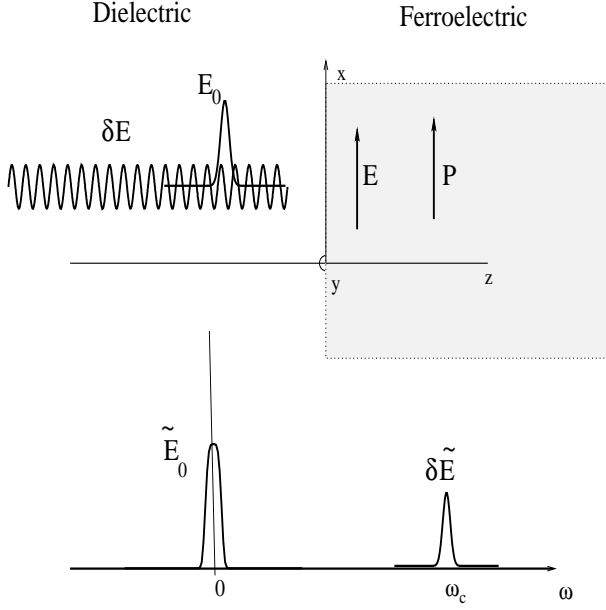


FIG. 1: Top panel: schematic drawing of the dielectric-ferroelectric interface. The electric field  $E$  is incident normally to the interface. It is polarized along  $x$  and the polarization is also along  $x$ . We show the control pulse  $E_0$  and the controlled small amplitude wave  $\delta E$  of frequency  $\omega_c$ , the carrier frequency (see text for details). The bottom panel indicates the Fourier spectra of these waves.

to be also along  $x$  and to depend only on  $z$ . Because of this the wave equation above reduces to

$$c^2 E_{,zz} - E_{,tt} = \frac{P_{,tt}}{\epsilon_0}. \quad (1)$$

In particular the term  $\nabla(\nabla \cdot \mathbf{P})$  is zero because the polarization does not vary along  $z$ , the direction of propagation of the pulse. The linear dielectric material ( $z < 0$ ) will be described by the Drude-Lorentz model [2] so that the polarization  $P$  follows

$$P_{tt} + \omega_0^2 P = \epsilon_0 \omega_p^2 E, \quad (2)$$

where  $\omega_0$  is the polarization frequency and  $\omega_p$  the plasma frequency. The polarization in the ferroelectric is given by the Landau-Khalatnikov equation [3]

$$\tau^2 P_{,tt} - AP + BP^3 = \epsilon_0 E, \quad (3)$$

where  $A = \alpha(T_c - T) > 0$  and  $B > 0$  are the Landau-Ginzburg coefficients and where  $\tau$  is the characteristic time of the polarization. The three equations (1,2,3) describe completely the field and polarization of the media.

For zero electric field, the spontaneous polarization of the ferroelectric is given by

$$-AP_0 + BP_0^3 = 0, \quad \rightarrow \quad P_0 = \pm \sqrt{\frac{A}{B}}. \quad (4)$$

Here the + (resp. -) sign is for a polarization in the direction of + $x$  (resp. - $x$ ). Now assume a field  $E_0$  constant during a time interval  $t_p$ . The polarization will then

shift and satisfy

$$-AP_0 + BP_0^3 = \epsilon_0 E_0. \quad (5)$$

For small  $E_0$  we can estimate  $P_0$  using  $E_0$  as a perturbation. We get

$$P_0 = \pm \sqrt{\frac{A}{B}} + \frac{\epsilon_0 E_0}{2A} + O(E_0^2). \quad (6)$$

Note that we did not consider the polarization close to 0 because it is unstable. During the time interval  $t_p$  we can send a small electromagnetic wave  $\delta E$ . This will shift the polarization by  $\delta P$ . Assuming  $E = E_0 + \delta E$ ,  $P = P_0 + \delta P$  where  $|\delta E| \ll E_0$ ,  $|\delta P| \ll P_0$  in equations (1,2,3) yields the linear system in the ferroelectric

$$c^2 \delta E_{,zz} - \delta E_{,tt} = \epsilon_0^{-1} \delta P_{,tt}, \quad (7)$$

$$\tau^2 \delta P_{,tt} - A \delta P + 3BP_0^2 \delta P = \epsilon_0 \delta E. \quad (8)$$

In the dielectric layer  $P_0 = 0$  so that the second equation should be replaced by

$$\delta P_{,tt} + \omega_0^2 \delta P = \epsilon_0 \omega_p^2 \delta E. \quad (9)$$

The three linear equations (7,9) represent the small oscillations  $(\delta E, \delta P)$  around the functioning point  $(E_0, P_0)$  which exists during the time  $t_p$ .

### III. SOLUTIONS OF THE WAVE EQUATIONS IN FREQUENCY DOMAIN

The system of linear equations above can now be solved completely using Fourier transforms in  $z$  and matching  $\delta E$  and its derivative at the interface  $z = 0$ . Taking the Fourier transform in  $z$  we get for  $z < 0$

$$\delta \tilde{E}_{,zz} + k_0^2 \delta \tilde{E} = -k_0^2 \epsilon_0^{-1} \delta \tilde{P}, \quad (10)$$

$$\delta \tilde{P} = \epsilon_0 \tilde{E} \frac{\omega_p^2}{\omega_0^2 - \omega^2}. \quad (11)$$

Plugging the second equation into the first one, we obtain

$$\delta \tilde{E}_{,zz} + k_0^2 \left( 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} \right) \delta \tilde{E} = 0. \quad (12)$$

In the ferroelectric medium for  $z > 0$ , starting from equations (7) and following a similar procedure as for  $z < 0$  we get

$$\delta \tilde{E}_{,zz} + k_0^2 \left( 1 + \frac{1}{-\tau^2 \omega^2 - A + 3BP_0^2} \right) \delta \tilde{E} = 0. \quad (13)$$

Substituting the expression (6) of the spontaneous polarization  $P_0$  we finally get

$$\delta \tilde{E}_{,zz} + k_0^2 \left( 1 + \frac{1}{2A \pm 3\epsilon_0 E_0 \sqrt{B/A} - \tau^2 \omega^2} \right) \delta \tilde{E} = 0. \quad (14)$$

Thus we have a piecewise wave equation for  $z < 0$  (12) and  $z > 0$  (14). In such linear media we can introduce the dielectric permittivity to describe wave propagation. We get

$$\varepsilon_{diel}(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}$$

in the dielectric layer and

$$\varepsilon_{ferr}(\omega) = 1 + \frac{1}{2A \pm 3\epsilon_0 E_0 \sqrt{B/A} - \tau^2 \omega^2}$$

in the ferroelectric layer. The solution in the two different regions is

$$E(z, t) = e^{i\omega t - ik_1 z} \quad z < 0, \quad E(z, t) = e^{i\omega t - ik_2 z} \quad z > 0$$

where the wave numbers  $k_1$  and  $k_2$  are

$$k_1 = k_0 \left( 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} \right)^{1/2}, \quad (15)$$

$$k_2 = k_0 \left( 1 + \frac{1}{2A \pm 3\epsilon_0 E_0 \sqrt{B/A} - \tau^2 \omega^2} \right)^{1/2}. \quad (16)$$

These formulas show us that the external electromagnetic pulse can control the dielectric properties of the ferroelectric material.

#### IV. SCATTERING OF LINEAR WAVES OFF THE INTERFACE $z = 0$ .

The reflection and transmission coefficients of harmonic waves can be computed as a function of the control field  $E_0$ . Note that the two orientations of polarization will give the same reflection coefficient for  $E_0 = 0$ . Only adding the control  $E_0$  is one able to distinguish the two states of polarization. We set up the scattering formalism assuming an incident wave from the left, a reflected wave and a transmitted wave,

$$\tilde{\delta}E(z, \omega) = E_{in}(\omega)e^{ik_1 z} + E_r(\omega)e^{-ik_1 z},$$

in the dielectric and

$$\tilde{E}(z, \omega) = E_t(\omega)e^{ik_2 z},$$

in the ferroelectric medium. In the absence of the surface charges and currents, the jump conditions on the interface read [4]

$$\tilde{E}(0-, \omega) = \tilde{E}(0+, \omega), \quad \tilde{E}_{,z}(0-, \omega) = \tilde{E}_{,z}(0+, \omega).$$

Using these conditions and the solution of the wave equation one can find the Fresnel relations connecting the

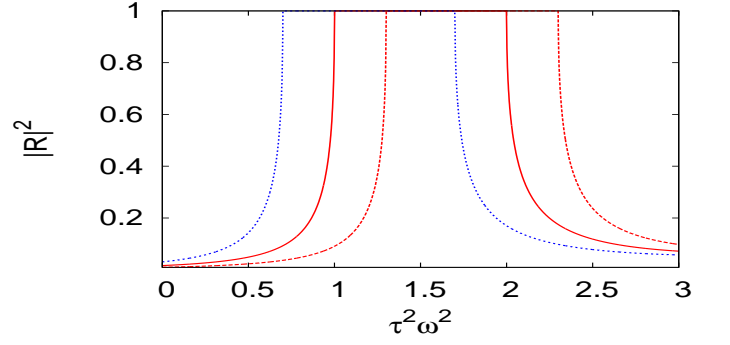


FIG. 2: Square of the modulus of the reflection coefficient  $|R|^2$  as a function of the reduced frequency  $\tau^2 \omega^2$  for three different values of the normalized control  $3\epsilon_0 E_0 / P_s = 0$  (continuous line, red online),  $0.3$  (long dash, red online) and  $-0.3$  (short dash, blue online). The parameters are  $A = 1$ ,  $B = 1$ .

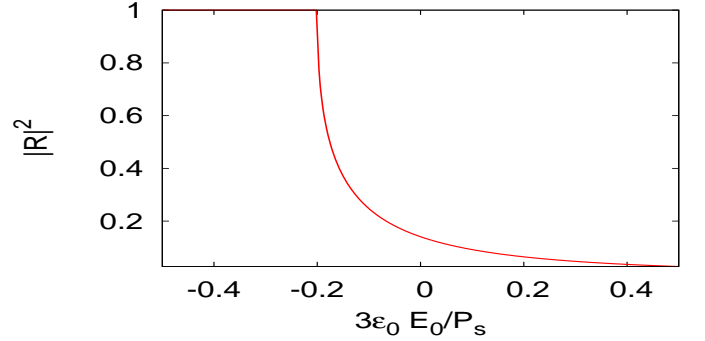


FIG. 3: Square of the modulus of the reflection coefficient  $|R|^2$  for a fixed reduced frequency  $\tau^2 \omega^2 = 0.8$  as a function of the normalized control  $3\epsilon_0 E_0 / P_s$ .

amplitudes of the incident wave  $E_{in}(\omega)$ , reflected wave  $E_r(\omega)$  and transmitted wave  $E_t(\omega)$ .

$$E_r(\omega) = \frac{k_1 - k_2}{k_1 + k_2} E_{in}(\omega), \quad (17)$$

$$E_t(\omega) = \frac{2k_1}{k_1 + k_2} E_{in}(\omega). \quad (18)$$

These relations are correct for any low amplitude waves, both solitary waves and for harmonic waves. The reflection and transmission coefficients are then respectively

$$R = \frac{E_r}{E_{in}} = \frac{k_1 - k_2}{k_1 + k_2}, \quad T = \frac{E_t}{E_{in}} = \frac{2k_1}{k_1 + k_2}, \quad (19)$$

#### V. DISCUSSION

Fig. 2 shows the modulus of the reflection coefficient  $|R|^2$  as a function of the reduced frequency  $\tau^2 \omega^2$  for three

different values of the control  $E_0 = 0$  (continuous line, red online),  $E_0 = 0.3$  (long dash, red online) and  $E_0 = -0.3$  (short dash, blue online). As can be seen the boundary of the gap for which there is total reflection of the wave is shifted to higher frequencies (resp. lower frequencies) for  $E_0 > 0$  (resp.  $E_0 < 0$ ). We have assumed the + sign in the expression of  $k_2$  (15). To illustrate how the field  $E_0$  can be used to block a wave we have plotted in Fig. 3  $|R|^2$  as a function of  $E_0$  for  $\tau^2\omega^2 = 0.8$ . For  $E_0 > 0.4$  the ferroelectric is transparent. As  $E_0$  is decreased  $|R|^2$  increases sharply and reaches 1 for  $E_0 = 0.2$ . Below that value the ferroelectric is opaque to this particular frequency.

To show how this scheme can work in reality, we examine parameters for a real material. Consider the study by Noguchi et al [8] on the  $Bi_4Ti_3O_{12}SrBi_4Ti_4O_{15}$  intergrowth ceramics. This material was shown to have a large spontaneous polarization. In addition its Curie temperature is high so that it is stable. To estimate  $A$  and  $B$  from the measurements of [8] we recall that

$$\frac{A}{B} = P_c^2, \quad \frac{2}{3}A\sqrt{\frac{A}{3B}} = \epsilon_0 E_c,$$

where  $P_c$  and  $E_c$  are respectively the coercitive polarization and coercitive field. Using these values from [8] we get

$$A = 210^{-3}, \quad B = 10^{-1}m^4C^{-2}.$$

This value of  $A$  defines a resonant frequency  $\omega_r$  such that

$$\tau^2\omega_r^2 = 2A.$$

We get

$$\tau\omega_r = 4.510^{-2}$$

The value of  $\tau$  given by estimates of the inertia of molecular assemblies is about  $\tau = 10^{-10}$  [9]. This gives

$$\omega_r = 4.510^8 Hz.$$

An important point is that at resonance the two terms  $2A$  and  $\tau^2\omega^2$  are almost equal so their difference is very small. Then a small shift due to the term  $3\epsilon_0 E_0 \sqrt{B/A}$  will displace the resonance. Let us estimate the field  $E_0$  needed to shift the resonance from  $\omega_r$  to  $\omega_r/2$ . We have

$$3\epsilon_0 E_0 \sqrt{\frac{B}{A}} = \tau^2 \frac{\omega_r^2}{4}.$$

This gives

$$E_0 \approx 10^6 V m^{-1}.$$

This value of the electric field can be achieved using a laser.

## VI. CONCLUSION

The reflection of the electromagnetic wave on the interface between linear dielectric medium and ferroelectric was considered. We assume that the electromagnetic wave is a superposition of a high frequency wave and a spike-like electromagnetic signal. The spectrum of the spike is located near the zero frequency and can be considered as low frequency. We showed that the spike-like signal induced an extra contribution to the total ferroelectric polarization. This causes a fast change of the reflection coefficient in the high frequency domain. In the time duration of the spike-like signal the relaxation processes can be neglected. Thus one can achieve a rapid control of light with light, using an extremely short, spike pulse.

- 
- [1] E.D. Mishina, N.E. Sherstyuk, V.I. Stadnichuk, A. S. Sigov, V. M. Mukhorotov, Yu. I. Golovko, A. van Etteger and Th. Rasing, Appl. Phys. Lett. **83**, 12 2402-2404 (2003)
  - [2] L. Rosenfeld, Theory of Electrons, New York: Dover Publications, 1965, pp. 68
  - [3] L.D. Landau, L.P. Pitaevskii, E.M. Lifshitz, *Electrodynamics of Continuous Media*, Elsevier Butterworth-Heinemann, Oxford, (2000).
  - [4] M.Born, E. Wolf, Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light, seventh (expanded) ed., Cambridge Univ. Press, Cambridge, UK, 2003.
  - [5] V.L. Ginzburg, JETP **15**, 739 (1945) (in Russian)
  - [6] A.F. Devonshire Philos. Mag. **40** 1040 (1949)
  - [7] V.L. Ginzburg Phys. Usp. **44** 1037 (2001)
  - [8] Y. Noguchi, M. Miyayama and T. Kudo, Appl. Phys. Lett., **77**, 3639-3641, (2000).
  - [9] J.-G. Caputo, A.I. Maimistov, E.D. Mishina, E.V. Kazantseva, V.M. Mukhortov, Phys. Rev. B **82**, 094113, (2010).